CHARACTERISTICS OF THE USE OF A HEAT-FLUX SENSOR IN INSTRUMENTS FOR DETERMINING THERMAL CONDUCTIVITY

T. G. Grishchenko

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We give the results of an analytic investigation of the effect of external conditions on the heat field of a specimen placed in a lambda apparatus carrying out the stationary plate method using a heat-flux sensor.

A method recently developed for the determination of thermal conductivity is the heat-flux sensor method [1-7], the essential feature of which is that information concerning the heat-flux density in a specimen is obtained directly from the readings of a sensor, and information concerning the temperature head is evaluated from the readings of thermocouples. The use of a heat-flux sensor in the instrument employed for the stationary plate method makes it possible to simplify the design of the instrument, the measurement technique, and the processing of the results, thus considerably shortening the experiment time [7].

For rigorous formulation of the experiment and high accuracy of the results obtained, the measured quantities must be independent of possible external disturbances. An analytic investigation of the effect of such disturbances reduces to the solution of a two-dimensional Laplace problem in cylindrical coordinates with boundary conditions of the third kind; on the end faces of the specimen, which are plane surfaces, there are specified the uniformly distributed thermal resistances R_1 and R_2, characterizing the heat exchange between the specimen and the adjacent walls, whose temperatures are t_1 and t_2, respectively:

$$\lambda \frac{\partial t(x,\ r)}{\partial x} = \frac{1}{R_1}(t_1 - t(x,\ r)),\quad x = h,$$

$$\lambda \frac{\partial t(x,\ r)}{\partial x} = \frac{1}{R_2}(t(x,\ r) - t_2),\quad x = 0.$$

On the lateral surface there is heat exchange with the surrounding medium, whose temperature t_m is assumed to be constant:

$$\lambda \frac{\partial t(x,\ r)}{\partial r} = \alpha(t_m - t(x,\ r)) \quad \text{for } r = R.$$

The problem thus formulated is equivalent in dimensionless coordinates to

$$\frac{1}{L^2}\frac{\partial^2\theta(\varkappa,\ \rho)}{\partial\varkappa^2} + \frac{\partial^2\theta(\varkappa,\ \rho)}{\partial\rho^2} + \frac{1}{\rho}\frac{\partial\theta(\varkappa,\ \rho)}{\partial\rho} = 0 \tag{1}$$

for $0 \leqslant \varkappa \leqslant 1$ and $0 \leqslant \rho \leqslant 1$;

$$\frac{1}{H_1}\frac{\partial\theta(\varkappa,\ \rho)}{\partial\varkappa} = \theta_1 - \theta(\varkappa,\ \rho) \quad \text{for } \varkappa = 1, \tag{2}$$

$$\frac{1}{H_2}\frac{\partial\theta(\varkappa,\ \rho)}{\partial\varkappa} = \theta(\varkappa,\ \rho) - \theta_2 \quad \text{for } \varkappa = 0, \tag{3}$$

$$\frac{\partial\theta(\varkappa,\ \rho)}{\partial\rho} + \mathrm{Bi}\,\theta(\varkappa,\ \rho) = 0 \quad \text{for } \rho = 1, \tag{4}$$

$$\frac{\partial\theta(\varkappa,\ \rho)}{\partial\rho} = 0 \quad \text{for } \rho = 0, \tag{5}$$

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A solution of this system obtained by the method of integral transforms [8] carried out with respect to the variable ρ, since the boundary conditions (4) and (5) are homogeneous, has the form

$$\theta(\varkappa, \rho) = \frac{2\mathrm{Bi}^2}{\alpha R_1} \sum_{n=1}^{\infty} \frac{J_0(\nu_n \rho)}{\nu_n J_0(\nu_n)(\mathrm{Bi}^2 + \nu_n^2)} [(D_{n,2} + D_{n,1}^{-1}) \operatorname{sh} \nu_n L + (1 + D_{n,2} D_{n,1}^{-1}) \operatorname{ch} \nu_n L]^{-1} [\theta_1 (\operatorname{sh} \nu_n L\varkappa + D_{n,2} \operatorname{ch} \nu_n L\varkappa) + \theta_2 (\operatorname{sh} \nu_n L(1-\varkappa) + D_{n,1} \operatorname{ch} \nu_n L(1-\varkappa))], \tag{6}$$

where $D_{n,j} = \nu_n \alpha R_j/\mathrm{Bi}$, $j = 1, 2$; the Bessel function $J_0(\nu_n L)$ is the kernel of the transform, and the eigenvalues ν_n are the roots of the equation $\nu J'_0(\nu) + \mathrm{Bi} J_0(\nu) = 0$.

For an analysis of the effect of the boundary conditions on the heat field of the specimen, we consider the function represented by the heat flux density in a direction perpendicular to the end faces of the specimen and averaged over the surface. Since the surface-averaged heat-flux density in this direction, written in dimensionless coordinates, is equal to $q_{av} = 1/\pi\rho^2$, making use of the partial derivative of the function (6) with respect to the variable $\varkappa$, we can write:

$$q_{\mathrm{av}}(\varkappa, \rho) = \frac{4L\mathrm{Bi}^2}{\alpha R_1} \sum_{n=1}^{\infty} \frac{J_1(\nu_n \rho)}{\nu_n \rho J_0(\nu_n)(\mathrm{Bi}^2 + \nu_n^2)} P_n(\varkappa),$$

where

$$P_n(\varkappa) = \left[\left(\frac{\alpha R_2 \nu_n}{\mathrm{Bi}} + \frac{\mathrm{Bi}}{\alpha R_1 \nu_n}\right) \operatorname{sh} \nu_n L + \left(1 + \frac{R_2}{R_1}\right) \operatorname{ch} \nu_n L\right]^{-1} \times \left[\theta_1 \left(\operatorname{ch} \nu_n L(1-\varkappa) + \frac{\alpha R_2 \nu_n}{\mathrm{Bi}} \operatorname{sh} \nu_n L\varkappa\right) - \theta_2 \left(\operatorname{ch} \nu_n L(1-\varkappa) + \frac{\alpha R_1 \nu_n}{\mathrm{Bi}} \operatorname{sh} \nu_n L(1-\varkappa)\right)\right].$$

Let us investigate the nature of the variation of the function $q_{\mathrm{av}}(\varkappa, \rho)$ along a radius by using a dimensionless function $\Pi(\varkappa, \rho)$, equal to

$$\Pi(\varkappa', \rho) = q_{\mathrm{av}}(\varkappa', \rho)/q_{\mathrm{av}}(\varkappa', \rho'), \tag{7}$$

where ρ' has a fixed value, e.g. $\rho' = 0.1$; $\varkappa'$ is any cross section along the height of the specimen. Since all the cross sections for the stationary problem under consideration are equivalent, we can simplify our calculations by setting $\varkappa' = 0$, which corresponds to the cross section at which the heat-flux sensor is situated.

The large amount of calculation required made it necessary to set up this problem on a computer. The program formulated on the Minsk-32 computer and realizing the expressions (6) and (7) provides for cycling of the calculations for all initial data and variable parameters. The initial temperatures t_1 and t_2 were varied in such a way as to obtain results for some characteristic regimes with respect to the specimen temperature (t_{sp} = –20, +20, 30, 60, 100°C) and the temperature gradient (grad t = 250, 500, 1000, 2000, 4000, 8000, 16,000 K/m).

The thickness of the specimen was taken to be equal to 1, 2, 4, 8, and 16 mm. The variation of the number Bi was specified by varying the thermal conductivity of the material, multiplying by successive factors of 10 (λ = 0.025, 0.25 and 2.5 W/(m·K)) for a constant heat-transfer coefficient equal to 5 W/(m²·K), which corresponds to the heat exchange by free convection at the lateral surface of the specimen. Some results of the calculation are shown in Figs. 1 and 2.

This investigation showed that the relative variation of the average heat-flux density along the radius of the specimen is almost independent of the specimen thickness and of the heat-transfer conditions on the lateral surface, provided that the average temperature of the specimen is equal to the temperature of the surrounding medium (see Fig. 1).

If the specimen temperature differs from the surrounding temperature only slightly, then, as can be seen in Fig. 2, the distortions in the heat field do not penetrate into the specimen beyond a value of ρ = 0.5. When the average temperature of the specimen is much higher or much lower, the distortions of the heat field become substantial and penetrate into the specimen to a considerable distance, reaching values of ρ = 0.3 for t_{sp} = 60°C and –20°C and ρ = 0.2 for t_{sp} = 100°C.

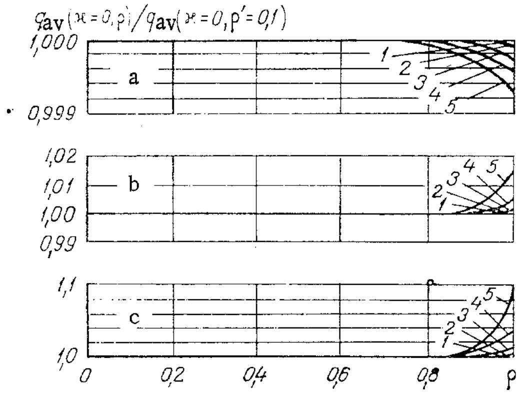

Fig. 1. Relative variation of the average heat-flux density as a function of the specimen thickness [1) h = 1 mm; 2) 2; 3) 4; 4) 8; 5) 16 mm] and of the conditions at the lateral surface [a) Bi = 0.1; b) 1.0; c) 10] for an average specimen temperature of 20°C.

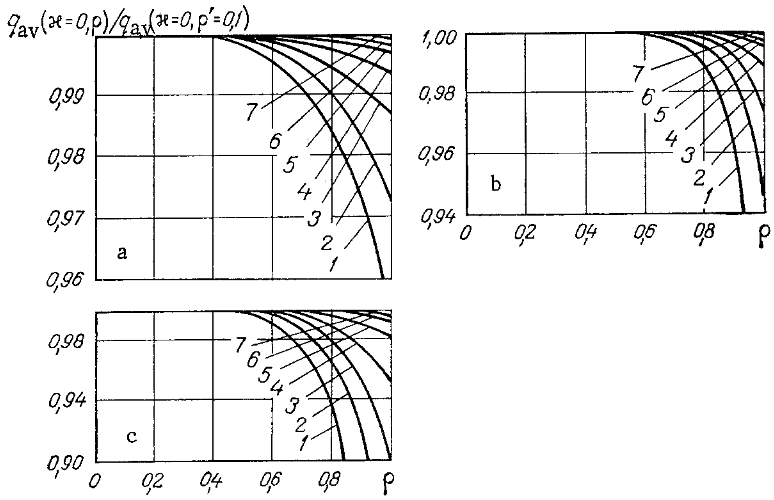

Fig. 2. Relative variation of the average heat-flux density as a function of the temperature gradient for t_{sp} = 30°C; 1) grad t = 250 K/m; 2) 500; 3) 1000; 4) 2000; 5) 4000; 6) 8000; 7) 16,000 (a — Bi = 0.1; b — 1.0; c — 10).

Thus, if the investigation of the thermal conductivity must be carried out over a working temperature range from –60°C to 100°C, then the sensitive element used for measuring the heat-flux density should be set up on a central area whose diameter should preferably be no more than 0.2 times the radius of the specimen.

The results described were obtained for thermal resistance values R_1 and R_2 equal to 0.005 and 0.007 $m^2 \cdot K/W$, respectively. The value of R_1 is determined by the thermal resistance of the temperature-measuring device placed between the specimen and the isothermal surface of the heat source. The quantity R_2 is composed of the thermal resistance of the second temperature-measuring device, placed between the specimen and the heat absorber, and the thermal resistance of the layer of sealing compound. This compound is applied to the calorimeter on the isothermal surface of the metal heat absorber. The value of the thermal resistance R_2 is assumed to be constant along the radius and equal to the thermal resistance of the heat-flux sensor (R_3).

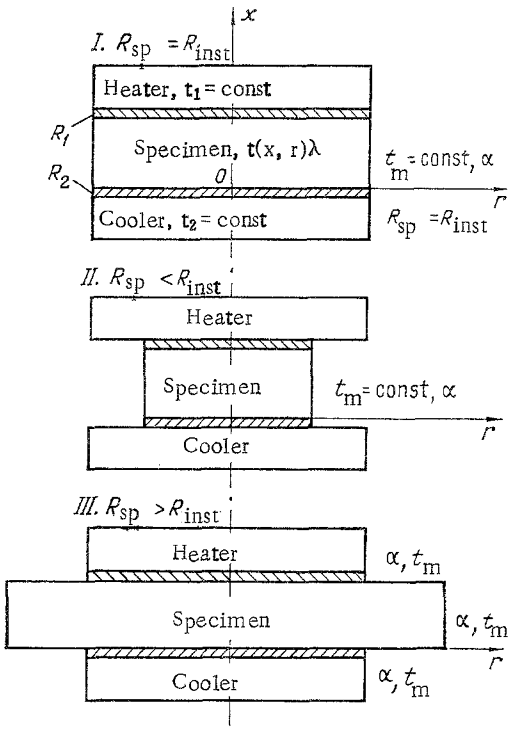

Fig. 3. Schematic arrangement of the specimen in the lambda apparatus.

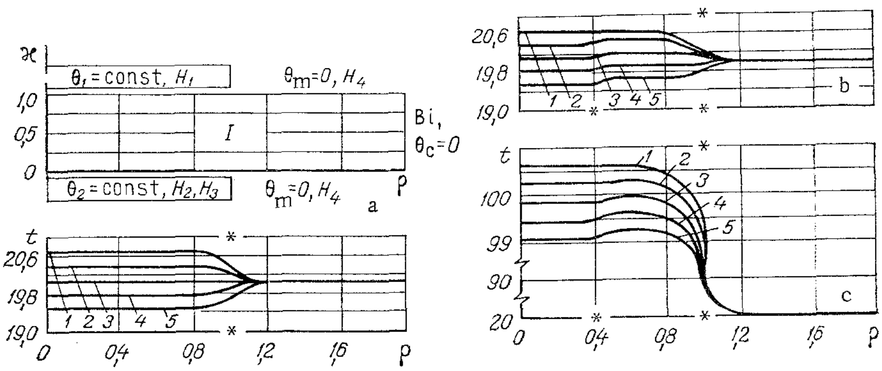

Fig. 4. Distribution of temperature over the cross sections of a specimen with discontinuous boundary conditions: 1) $\varkappa$ = 1; 2) 0.75; 3) 0.50; 4) 0.25; 5) 0; I is the region of finite elements, 4 × 50.

In addition to the case considered above, when the diameter of the specimen is equal to the diameter of the working segment of the lambda apparatus, it is also of interest to consider two other cases, in which these diameters are unequal (see II and III in Fig. 3).

When the specimen is smaller, the temperature field of the specimen is described by a system analogous to (1)-(5), and therefore all the results obtained in solving it can be extended to specimens of this type. From a comparative analysis it follows that the minimum diameter admissible for the specimen investigated at a temperature close to room temperature, i.e., in the interval (t_m ± 10), must be twice as large as the diameter of the heat-flux sensor used in the instrument.

If the specimen goes beyond the limits of the instrument, as shown in Fig. 3, III, the boundary conditions on the end faces are considerably altered, and in dimensionless form we can write them as follows:

$$\frac{\partial\theta(\varkappa=1,\ \rho)}{\partial\varkappa}=\begin{cases} H_1(\theta_1-\theta(\varkappa=1,\rho)) & \text{for}\quad 0\leqslant\rho\leqslant 1,\\ -H_4\theta(\varkappa=1,\rho) & \text{for}\quad 1<\rho\leqslant\bar{\rho};\end{cases} \tag{8}$$

$$\frac{\partial\theta(\varkappa=0,\rho)}{\partial\varkappa}=\begin{cases} H_2(\theta(\varkappa=0,\ \rho)-\theta_2) & \text{for}\quad 0\leqslant\rho\leqslant 1, \\ H_4\theta(\varkappa=0,\ \rho) & \text{for}\quad 1<\rho\leqslant\bar{\rho}, \end{cases} \tag{9}$$

where $\bar{\rho} = R_{sp}/R_{inst} > 1$.

As we can see, the boundary conditions in (8) and (9) are expressed by piecewise-continuous functions. They involve the partial derivative of the temperature $\theta(\varkappa,\rho)$ with respect to the variable $\varkappa$ for $\varkappa = 0$ and $\varkappa = 1$, which has finite discontinuities at $\rho = 1$, i.e., at the point where the specimen projects from the instrument. This makes it difficult to obtain an analytic solution.

In order to find the numerical solution of the problem, we used the LAPLAS program, developed at the Moscow Scientific-Research Institute of Prototype and Experimental Design. In accordance with this program, the specimen is represented in the form of a region of finite elements, for whose nodes we obtain temperature values on the computer. The calculation was carried out for the Laplace equation (1) with the following boundary conditions: (4) for $\rho = \bar{\rho}$; (5) for $\rho = 0$; (8) for $\varkappa = 1$; and (9) for $\varkappa = 0$. When the layer in which the heat-flux sensor is placed consists of two segments whose thermal resistances are not equal, another finite discontinuity must be introduced into the condition (9):

$$\frac{\partial\theta(\varkappa=0,\rho)}{\partial\varkappa}=\begin{cases} H_2(\theta(\varkappa=0,\ \rho)-\theta_2), & 0\leqslant\rho\leqslant\bar{\bar{\rho}}, \\ H_3(\theta(\varkappa=0,\ \rho)-\theta_2), & \bar{\bar{\rho}}<\rho\leqslant 1, \\ H_4(\theta(\varkappa=0,\ \rho)), & 1<\rho<\bar{\rho}, \end{cases}$$

where $\bar{\bar{\rho}} = R_{hfs}/R_{inst} < 1$.

Figure 4 shows the results of the calculation for a specimen 200 mm in diameter and 4 mm in thickness, with a thermal conductivity of 0.25 W/m·K. Figure 4a shows the temperature-distribution functions for five cross sections along the height of the specimen for an average temperature of 20°C when there are discontinuities in the boundary conditions only for $\rho = 1$; Figs. 4b and c show analogous functions for an average specimen temperature of 20°C and 100°C, respectively, taking account of the discontinuity at the cross section $\varkappa = 0$ when $\rho = \bar{\bar{\rho}}$.

From an analysis of the resulting temperature fields it follows that the piecewise-continuous nature of the boundary conditions results in a distortion of the fields in the vicinity of the points of discontinuity, but it has no effect on the uniformity of the heat flux in the central part of the specimen. The difference between the thermal resistances of the heat-flux sensor and the surrounding layer of sealing compound affects the uniformity of the heat flux being measured. In order to eliminate this effect, a zone whose thermal resistance is equal to that of the heat-flux sensor is set up around it.

Thus, the heat flux being measured by the sensor is uniform in the central zone of the specimen (not more than 0.2 radii) when it is subjected to the possible thermal effects of external conditions in the temperature and temperature-gradient ranges under consideration. As a result, in determining the thermal conductivity of a material by the heat-flux sensor method on a specimen whose diameter is such that the flux passing through the specimen and measured by the sensor set up in the instrument is one-dimensional, it is possible to use the well-known relation $\lambda = qh/\Delta t$ obtained for the one-dimensional heat-conduction problem.

NOTATION

R_1, R_2, R_3, thermal resistance; t, temperature; t_m, temperature of the medium; t_{sp}, temperature of the specimen; λ, α, thermal conductivity and heat-transfer coefficient; x, r, space coordinates; h, R, thickness and radius of the specimen; q, heat-flux density; $\varkappa = x/h$, $\rho = r/R$, $\theta = (t - t_m)/t_m$, dimensionless coordinates; $H_j = h/(\lambda R_j)$, j = 1, 2, 3, $H_4 = \alpha h/\lambda$, $L = h/R$, dimensionless parameters; $Bi = \alpha R/\lambda$, Biot number; J_0, J_1, Bessel functions of the first kind, of orders 0 and 1; ν_n, eigenvalues.

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THERMAL SIMULATION OF ELECTRICAL DEVICES

G. N. Dul'nev, B. V. Pol'shchikov,
and O. B. Aga

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A classification of a wide class of electrical instruments is presented, their thermal and mathematical models are formulated, and a "thermal mode" subsystem is considered in a general system of computer-aided design.

The modern development of power engineering, machine construction, radio engineering and other areas of industry requires a greater and greater use of high-power electrical equipment — power and transformer devices, rectifier units, power supplies and other devices based on high-power thyristors, diodes, and transistors. The development of such apparatus involves solving a number of problems, one of which is to ensure the normal thermal operating conditions. The increasing rate of increase in production, the increasing complexity of the construction of the devices, and the reduction in the time available for development require solutions of design problems using computer-aided methods, where the thermal design must be regarded as a subsystem in the overall design system. To set up such a subsystem it is necessary to solve a number of problems of which one of the main ones is to develop thermal and mathematical models for the devices considered and to realize them in practice.

Hierarchical Principle of Layout. For a systematic approach to the design, the individual electrical devices or apparatus are considered as a whole, their characteristic components are distinguished, and the relations between them are studied [1]. Optimum design of the device architecture is carried out with a further stage-by-stage optimization of the constructional units. The systematic approach is based on the hierarchical principle of modeling, which is considered in detail below.

The main circuits and construction of high-power and transformer equipment have been considered fairly completely in [2-7], and analysis of this enables us to distinguish the following hierarchical levels.

The first level includes constructionally completed elements and components of the construction, viz., transistors, thyristors, diodes, and other devices. The devices of the second level are typical replacement elements, the main constructional units (modules), which combine one or several semiconductor elements 1, together with an individual or group 2 of cooling systems. Examples of the use of different forms of cooling — air, liquid, and evaporative, are shown in Fig. 1a [3-7].

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